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On the Embedded Complementarity of Agent-Based and Aggregate Reasoning in Students' Developing Understanding of Dynamic Systems

Walter M. Stroup · Uri Wilensky

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Abstract Placed in the larger context of broadening the engagement with systems dynamics and complexity theory in school-aged learning and teaching, this paper is intended to introduce, situate, and illustrate—with results from the use of network supported participatory simulations in classrooms—a stance we call 'embedded complementarity' as an account of the relations between two major forms of systems-related learning and reasoning. The two forms of systems reasoning discussed are called 'aggregate' and 'agent-based.' These forms of reasoning are presented as distinct yet we also outline how there are forms of complementarity, *between* and *within* these approaches, useful in analyzing complex dynamic systems. We then explore specific ways in which the embedded complementarity stance can be used to analyze how learner understandings progress in science, technology, engineering, and mathematics-related participatory simulations supported by the HubNet (Wilensky and Stroup 1999c) learning environment developed with support from the National Science Foundation. We found that the learners used and built on the interdependence of agent and aggregate forms of reasoning in ways consistent with the discussion of embedded complementarity outlined in the early parts of the paper.

Keywords Dynamic systems · Complexity theory · Agent-based modeling · Aggregate modeling · Participatory simulations · NetLogo · HubNet

Framed in terms of broadening the engagement with systems dynamics and complexity theory in school-aged learning and teaching, this paper is intended to introduce, situate, and illustrate—with results from the use of network supported participatory simulations in classrooms—a stance we call 'embedded complementarity' as an account of the relations

W. M. Stroup (✉)
The University of Texas at Austin, Austin, TX, USA
e-mail: wstroup@austin.utexas.edu

U. Wilensky
Northwestern University, Evanston, IL, USA

between two major forms of systems-related learning and reasoning. The two forms of systems reasoning, to be discussed at greater length in subsequent sections of this paper, are called ‘aggregate’ and ‘agent-based.’

With funding from the National Science Foundation (NSF) we have developed an open architecture [*HubNet*, Wilensky and Stroup (1999c)] for authoring and implementing highly interactive, group-oriented, network-supported activities called participatory simulations. This architecture allows us to explore aspects of the development of forms of systems reasoning with school-aged learners. Within this immersive environment, learners can assume the role of rabbit or wolf in a predatory-prey simulation, explore algebraic ideas by becoming movable points in a Cartesian coordinate system, or explore challenges related to civil engineering by controlling individual traffic lights in a simulated traffic grid and then attempt to work together to develop strategies for improving overall traffic flow.

Embedded complementarity emerges as a way of understanding how to engage and advance understandings of system dynamics and complexity theory in relation to learning and teaching important and challenging concepts in science, technology, engineering, and mathematics (STEM) domains. Our intent is that this paper might provide enough background and detail from classroom implementations to support fellow researchers or school-based educators in attending to and exploring the utility of this framework for characterizing and advancing systems frameworks within an effort to significantly improve the quality, significance, and diversity of participation in STEM-related fields of inquiry and on-going civic engagement.

The goal of having school-aged learners investigate ideas related to systems dynamics and complexity theory has been advanced in the United States, somewhat episodically in cycles of every 10–15 years, in various national standards documents or curricula since at least the advent of the Science Curriculum Improvement Study (SCIS) in the early 1960s (Karplus 1964). Sometimes, as with SCIS in the immediate post-Sputnik era, these efforts were explicitly tied to broad efforts to reorganize and reform approaches science learning and teaching. At other times the calls emerge in relation to the development of new computational modeling tools and approaches, like the use of an iconic interface for creating and implementing finite difference models [e.g., *VenSim* (Ventana Systems 1995), *STELLA* (Richmond and Peterson 1984, 1990) or *InSight Maker* (Foreman-Roe and Bellinger 2013)] within a system dynamics framework (cf., Forrester 1993) or the creation and use of agent-based computer modeling [e.g., *NetLogo* (Wilensky 1999a); *StarLogo*, (Resnick and Wilensky 1993; Wilensky 1993; Resnick 1994; Wilensky and Resnick 1999) or AgentSheets (Repenning 1993)] as a more embodied modeling approach.

By linking a broad and inclusive sense of STEM literacy with having all students develop an understanding of the idea of a “system,” the *Benchmarks for Science Literacy* developed by the American Association for the Advancement of Science (1993) is typical of many of the prior reform efforts in stating: “One of the essential components of higher-order thinking is the ability to think about a whole in terms of its parts and, alternatively, about parts in terms of how they relate to one another and to the whole” (AAAS 1993). Even the more recent *National Science Education Standards* allows, as one of the goals, that students be able to “think and analyze in terms of systems” as a strategy for responding to the “natural and designed” world being much “too large and complicated to investigate and comprehend all at once” (NRC 1996, p. 119).

Like many of these prior efforts, we are committed to a stance suggesting the study of systems-related ideas is crucial not only to the progress of science but also to the understanding of ideas related to feedback, emergence, or how chance or random interactions can lead to well-ordered overall outcomes—as just three examples—might play a constructive role in supporting civic participation in a democracy and a sense of personal agency in being able to act in an increasingly complex and dynamically-defined world. Also like some of the previous efforts, the work reported on herein is occasioned, in part, by the development and use of a new kind of network-based, interactive, computation environment—*HubNet* (Wilensky and Stroup 1999c) built on top of NetLogo—as well as immersive, group-oriented, activities called "participatory simulations" implemented in this environment.

What might be distinctive in the work reported on herein is: (1) our introduction and situating of "embedded complementarity" as an account the inter-relationships between two major forms of systems-related reasoning and (2) the presentation of a relatively detailed, if still somewhat preliminary, set of analyses illustrating the utility of embedded complementarity in accounting for how systems-related actions and understandings develop when STEM-specific, HubNet supported, participatory simulations are used in classrooms. We begin by framing our efforts in terms of prior systems learning research and then clarify what we mean by embedded complementarity. A description of the HubNet architecture is provided. Then, in the final sections of the paper, embedded complementarity is deployed as a framework for analyzing instances of classroom-situated, systems-related learning and teaching using participatory simulations implemented in the HubNet architecture.

1 Prior Related Systems Learning Research

As briefly noted earlier, the literature related to systems learning, systems learning projects and systems learning tools extends from at least the 1960s to the present (e.g., Karplus 1964; Blauberg et al. 1977; Chen and Stroup 1993; Doyle 1997; Forrester 1961, 1968, 1993; Garigliano 1975; Hestenes 1995; Jackson et al. 1996; Jacobson et al. 2011; Lemke 2000; Levy and Wilensky 2008; Mettes 1987; Mandinach and Cline 1994; Mandinach and Thorpe 1987; Ogborn 1985, 1996; Ogborn and Wong 1984; Richmond and Peterson 1984, 1990; Roberts 1978; Roberts et al. 1983; Senge 1990; Starr 1994; Steed 1992; Resnick 1994, 1996; Wilensky 1993, 2001; Wilensky and Jacobson in press). Taken together, these efforts have developed a variety of computer-based tools, curricula, models for teacher development, and important insights related to learning and teaching about system dynamics and complexity theory. This progress notwithstanding, significant issues remain related to learners developing nuanced and expressive systems-related abilities and insights.

Some of the learning-specific challenges overlap with what might be seen as 'classic' developmental issues related to issues of conservation of quantity and kind, but now extended to the context of making sense of system behavior (cf., Garigliano 1975). Others seem to implicate learners' understandings of the dynamics of rate and amount (Stroup 1993, 1996, 2002, 2005; Wilkerson-Jerde and Wilensky 2009), effective points of entry into agent-based modeling environments (Levy and Wilensky 2008; Wilensky 1997, 1999b, 2003), the ability to effectively incorporate experiential, first-person perspective, learning (Resnick and Wilensky 1998), the access to and intelligibility of models (Kay 1991; Starr 1994; Wilensky 1995, 1996, 2001; Wilensky and Reisman 1998, 2006), and integration with current curriculum (Zaraza and Fisher 1996; Gobert et al. 2003).

The more recent analyses of systems learning tend to reflect a movement from prior developmental and/or constructivist learning frameworks, to much more computationally-structured accounts of cognition and expertise found in the still nascent 'learning sciences.'

For this recent work the contrast between novice and expert performance—having its roots in cognitive psychology and then in efforts to develop expert systems within artificial intelligence research—now serves as a defining feature of many investigations carried out in the learning sciences [e.g., *How People Learn* (HPL), Bransford et al. (2000), Chap. 2, pp. 31–50). Consistent with this movement away from other frameworks for analyzing learning and teaching, important recent syntheses from within the learning sciences have looked to provide specific accounts of “novice and expert” understandings of “complex systems” (Hmelo-Silver and Pfeffer 2004, see also, Chi et al. 2012; Jacobson et al. 2011).

Perhaps reflective of our own extended involvement in systems theory research (cf. Chen and Stroup 1993), we find ourselves somewhat less committed, at least in the near term, to framing our work in terms of transitioning from novice to expert performance. We understand that much of the power and warrant of the novice-expert framework, as deployed within the learning sciences, flows precisely from a commitment to being able to move readily between characterizing—*qua* experts and in contrast with novices—the abilities of both “sushi experts” [Hatano and Inagaki (1986) referenced in HPL, p. 45] and those of research physicists [Chi et al. (1981) referenced in HPL, p. 41]. Nevertheless, our own work with system dynamics and complexity theory tends to center less on global novice-expert accounts and much more on a notion of big or “powerful ideas” (e.g., Papert 1980) that STEM-oriented communities of practice will tend to consistently highlight as central to their on-going sense making activities. From within these overlapping and relatively longstanding communities of STEM practice, such big ideas tend not to be viewed simply as “facts” or forms of propositional knowledge to be drawn on by experts (or entered into the code for an expert system), but instead are ways of participating in, and assuming agency relative to, the world of dynamic, lived experience.

In our analyses, then, we limit ourselves to a comparatively modest effort to understand and simultaneously extend aspects of what has worked over time—and in ways that are likely to continue to work—for the development and advancement of highly social and historically situated activities associated with STEM domains. Working within a socially situated and participatory account of what it means to learn and teach STEM-related practices (cf., Ares et al. 2009 for a related account), we attempt to attend to the emergence of ideas that seem to matter in relation to on-going learning and teaching in STEM-focused classrooms. Embedded complementarity is simply a name given to forms of systems-related reasoning we suggest are of importance within STEM domains and, closely related, in relation to forms of expression and participation exhibited in classrooms.

2 What is Meant by Embedded Complementarity

If we are to advance the claim that embedded complementarity (EC) is a useful framework for pursuing system dynamics and complexity theory in relation to STEM curricula and classroom-based learning and teaching, we first need to address two related preliminary questions: (1) What are aggregate and agent-based forms of reasoning? and (2) What do we mean by suggesting that embedded complementarity characterizes important aspects of how these forms of reasoning might be seen as related? Using the example of the spread of a disease in a finite population, we will discuss these questions so as to help frame the sorts of expressions, gestures and interactions we will attend to more carefully in the subsequent analyses of classroom-based learning and teaching. A participatory simulation about the spread of a disease is one of the activities we discuss in subsequent sections.

Although our intent is to introduce embedded complementarity in a way that frames the accounts of using the HubNet architecture to support the use of STEM-related participatory simulations, we should make clear from the start that the position we take herein is not meant to be fully exclusive of other positions and their established warrant for investigating systems related learning and teaching. Rather the goal is simply to outline aspects of the EC stance.

2.1 What are Aggregate and Agent-Based Forms of Systems Reasoning?

Broadly speaking, there are two widely recognized and frequently deployed approaches to formally modeling dynamic systems that we use to frame our analyses. *Aggregate modeling* has come to be associated—at least since Isaac Newton's seventeenth century use of the flow-oriented language of *fluxion* and of *fluent* in developing his mathematic of change (Boyer 1959, p. 194)—with "rate" and "amount" or "flows" and "stocks" (Forrester 1961, 1968, 1993), and includes calculus, differential equations, finite-difference and related numerical approaches, and where sometimes "macro" language and features are highlighted. *Agent-based modeling,* in contrast, begins with elements or agents in a system enacting local rules and interacting in ways where bottom-up, event-driven, individuals-based, "micro" language and features are highlighted.[1] (See also Levy and Wilensky 2008 and Wilkerson-Jerde and Wilensky 2009, 2010 for further discussions of these forms of reasoning).

2.2 What is Meant by Embedded Complementarity?

In previous work we've advanced the idea that "the ability to relate individual and aggregate behavior is crucial for understanding complexity" (Chen and Stroup 1993, p. 457). Our current use of the phrase "embedded complementarity" is meant to update, and in some ways clarify, what we mean by an ability to "relate" agent-based and aggregate behavior.

An example similar to one of the participatory simulations discussed later may help to clarify aspects of what is meant by embedded complementarity. Figure 1 presents two accounts—one aggregate (left side) and one agent-based (right side)—of the spread of a disease in a population of forty-one individuals or agents.

The account on the left side of the figure foregrounds aspects of aggregate reasoning in that the movement of individuals from the healthy population, to the sick population, is treated as a kind of flow (rate) that accumulates in the reservoir of sick individuals (amount). The rate of flow—represented by a valve icon on the pipe-like connector leading from the healthy reservoir to the sick reservoir—is related to the number well and the number sick in the population. How this relationship between the number well and the number of sick "works" is often what learners will want to discuss, or will be asked about, after controlling their icons in a round of playing the disease participatory simulation (Part Sim) (Wilensky and Stroup 1999b; Stroup and Wilensky 2002). Details of these discussions are to be taken up in subsequent sections.

[1] Examples of current or historically significant aggregate modeling environments include *STELLA* (Richmond and Peterson 1984,1990), *Model-It* (Jackson et al. 1996); Link-It (Ogborn 1984), VenSim (Ventana Software 1995), *Insight Maker* (Foreman-Roe and Bellinger 2013). Current or historically significant agent-based modeling environments include *NetLogo* (Wilensky 1999a); *StarLogo* (Wilensky and Resnick 1999); *Agentsheets* (Repenning 1993), *Swarm* (Langton and Burkhardt 1997), *Repast* (Collier and North 2011) and *Mason* (Luke et al. 2005).

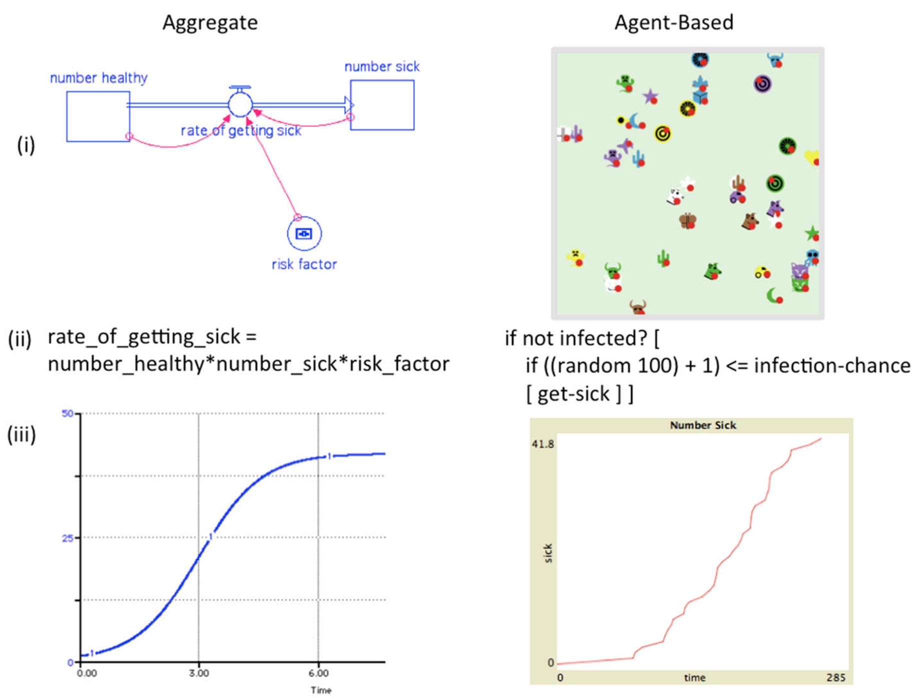

Fig. 1 Side-by-side comparison of (*i*) interfaces, (*ii*) respective samples of code, and (*iii*) the resultant graphs of the number sick for aggregate (*left*) and agent-based (*right*) models for the spread of a simulated disease

The right hand side of Fig. 1 also depicts the spread of a disease in a population of forty-one individuals, but now with the agent-based way of reasoning foregrounded. Each agent, represented by a distinctive icon capable of independent movement around the screen, starts off healthy. Then one individual agent gets "sick"—represented by a red dot appearing on the icon—and the disease spreads to other agents through their having contact with a sick agent (in the image shown in Fig. 1 all the agents have this red dot, meaning they are all infected). The depiction is agent-based because the individual members of the population are overtly depicted and because these agents can enact local rules for movement and interaction resulting in the bottom-up, event-driven spread of a disease as an instance of an emergent phenomenon.

As is suggested by the differences in the how the systems are presented, approaches that highlight either aggregate or agent-based accounts of the spread of a disease are distinct. Not only are the ways in which the icons refer to the phenomena distinct (stocks and flows versus agents moving about), but their more formal textual depictions are distinct as well. To illustrate this, excerpts from the working code for each model are presented in the figure next to each other (see (ii), below the iconic representations). The code excerpt from the aggregate model refers to "rate" and "number" (amount) whereas the code excerpt from the agent-based model (Wilensky and Stroup 1999b) is a rule for how an agent becomes sick.

From within the embedded complementarity stance, however, the fact that the approaches have clear distinctions does not require that these distinctions need be seen to mark out a kind of "incompatibility" (e.g., Chi 1992, 2000, 2005; Slotta 2011; Slotta and Chi 2006; Chi et al. 2012) in how experts might understand the spread of a disease or, indeed, in how aggregate and agent-based forms of reasoning might be productively juxtaposed in understanding a range of complex dynamic systems including the participatory simulations discussed herein. The observation that the graphs for the spread of the simulated disease were independently generated and yet are so similar (see Fig. 1), combined with the observation that both graphs are consistent with our expectations for how the spread of a disease might be expected to take place, point us toward the sense of complementarity we intend.

Analogous in some ways to how the complementarity between wave and particle accounts of matter work together in physicists' representations of the natural world, our use of complementarity is not to be seen as a placeholder for not yet having worked out either a fully-aggregate based way or a fully-agent based way of representing all that we might want to know about complex dynamic systems like the spread of a disease. Just as physicists use *both* wave and particle accounts of the world and where—despite the clear distinctions between each of these—we do not need to, *nor can we*, ultimately answer the question of which account is how the material world really works, we would suggest aggregate and agent-based ways of reasoning can and do work together, and can be viewed as complementary, in interrogating our experiences and in developing robust understandings of complex dynamic systems.

In addition to our wanting this sense of complementarity to forestall any expectation that either aggregate or agent-based forms of reasoning will ultimately be recast in terms of, be reduced to, or be replaced by the 'other,' the word "embedded" is added to highlight another feature of how the two work together in a way not anticipated by the use of complementarity within the domain of physics. Just as we hope to illustrate in the empirical portion of this paper how the juxtaposition of agent-based and aggregate forms of reasoning does occur and is productive in learners' on-going sense making related to the use of STEM-related participatory simulations in classrooms, we also want to be able point to instances of reasoning that illustrate a kind of embedded-ness, or conceptual interdependence, of agent-based and aggregate reasoning.

'Embedded' is therefore added to 'complementarity' to further clarify, or characterize, the (sometimes only tacit) relations and interactions *between* and *within* the respective forms of modeling. The parallelism suggested by the production of similar graphs (Fig. 1) can illustrate the sense of moving "between" dynamic agent-based modeling and dynamic aggregate modeling. What remains is for us to provide an overview of some illustrative examples from "within" each.

Within agent-based modeling we routinely (if often only tacitly) work with expectations about how 'space' is to function. A command asking agents in the *NetLogo* modeling environment to move *forward 5*, for example, requires a set of formalized, system-wide, expectations about how space is to work that are not defined locally by the agents themselves. Similarly, asking the agents to turn right *random 180* has implicit, in an embedded sense, formal expectations about how random is to work, to say nothing of what it should mean to turn right some number of degrees, that are not localized to the agents but are present in the rules the agents are to execute. We could even suggest the idea of counting (e.g., the number of sick agents in Fig. 1) and constructing graphs or histograms are ways of rendering what is happening in an agent-based model that cannot be localized, as forms of reasoning, 'in' the local interactions of the agents themselves.

These more-than-simply-local ideas are implicitly present, in what we are suggesting is a plausibly constitutive sense, in what it should be possible for agents to do, or to be, as computational objects. For an agent-based model of an ideal gas [e.g. *GasLab*, (Wilensky

1999b)], the addition of a plot of the root-mean squared (rms) of the velocities of the simulated molecules over time presents an invitation to define temperature, and then discuss 'its' rate of change (in 'time') when a simulated piston 'compresses,' 'adiabatically,' the simulated 'gas.' The quotes have been added as a further invitation to the reader to consider how constructs can become, or simply are, meaningful *aggregate* (or 'macro') constructs deployed in developing a more nuanced understanding of the thermodynamics of an ideal gas from *within* an agent-based model.

Within aggregate modeling some aspects of the referencing of agent-based, or what can be called 'micro,' accounts are present. When, for example, "*dN/dt*" is used in a differential equation related to population dynamics, the "N" is taken to reference the number, or population, of a certain species of animal. "N" is taken as plausibly referring to individuals, or agents, sufficiently similar as to be treated as a homogeneous collection. Consequently, dN/dt can then refer to the rate of change of that population in time. Similarly ΔU, or the change in internal energy of an object or sample, is routinely taken to reference changes in formalized accounts of the rates of movement, spins or vibrations, of molecules to allow for aggregate analyses. Probabilistic accounts from quantum mechanics can describe the likelihood of observing the radioactive decay of an atom, and thereby allows reference 'back' to our asking what can we say will happen to 'this' molecule. The discrete "clicks" of a Geiger counter corresponding to the atoms' decay can be described temporally in ways that can, in turn, be used to establish, or even negate, the veracity of an aggregate, probabilistic, set of claims regarding a given sample of a radioactive isotope.

These lists of embedded attributes are far from exhaustive and should be seen only to begin to draw out the ways in which, while learners might start out, or might even remain, modeling with one of the frameworks or approaches foregrounded, the interdependences are present and available. Expert and learner accounts might be expected to acknowledge and build on this interdependence, and these are the forms of reasoning we will attend to in our analyses of classroom-based learning episodes.

Finally, with the development of various technologies, as discussed earlier, helping to mark out some of the episodes in the history of dynamic systems modeling, we can find further support for the embedded complementarity account in the ways in which some of the modeling environments have moved to make more explicit and available some of the complementary aspects of agent-based or aggregate modeling. For example, *NetLogo,* with a clear object-oriented or agent-based pedigree, now has incorporated finite difference capabilities and flow icons similar to those found in *STELLA* [used to create the aggregate model of the spread of disease in Fig. 1, (Richmond and Peterson 1984, 1990)], *VenSim* (Ventana Systems 1995), or *InSight Maker* (Foreman-Roe and Bellinger 2013). Similarly *InSight Maker,* an environment with a clear aggregate pedigree, has incorporated significant support for agent-based modeling in recent versions.

Thus while agent-based and aggregate forms of modeling are clearly distinct, in this section we have outlined some of our reasons for suggesting they are also far from 'incompatible.' To reframe for this context Seymour Papert's observation that any particular representational approach should be viewed as "on tap, but not on top" [he was discussing the role of "text" in the context of doing mathematics (Papert 1998)], both aggregate and agent-based approaches are "on tap" without either being "on top" in any necessary or exclusive sense. With this framing of what is meant by embedded complementary as a way of inter-relating agent-based and aggregate forms of systems reasoning, we now move to discuss aspects of participatory simulations—especially as enacted using interactive, network technologies—as a kind of socially situated activity for supporting and advancing systems learning in group learning contexts, including classrooms.

3 Participatory Simulations and Network Supported Learning

Possibly the first major instance of where a participatory simulation was used in the context of systems dynamics and systems learning was *The Beer Game* as developed by Jay Forrester and his system dynamics group at MIT in the early 1960s. These participatory simulations were called "flight simulators," in a way that alluded to the military use of simulator environments in WWII. There is a significant literature related to *The Beer Game*, and interest in this participatory simulation was revitalized as a result of its appearance in Peter Senge's widely read *The Fifth Discipline* (1990). Diehl (1990) appears to have been the first to use the phrase "participatory simulations" to describe these activities.

From the 1960s onward, different technologies have been used to implement participatory simulations. These implementations range from the use of simple paper and pencil (e.g., Senge 1990; Stor and Briggs 1998), to the use of electronic badges [so-called "Thinking Tags," see Colella 1998; Colella et al. 1998; Borovoy et al. 1996), and handheld technologies (Stroup 1997 (using a TI-83 graphing calculator); Soloway et al. 2001 (using a Palm OS device)]. Because of their power in making thinking about the emergence and development of systems thinking visible to the teacher, students and researchers alike, our focus in this article is on participatory simulations as implemented in agent-based modeling environments and next-generation classroom networks.

Next-generation network systems for supporting participatory learning are designed specifically for group learning and teaching. Rather than simply import traditional network capabilities associated with business or other less learning-centered environments, these systems are specifically optimized for classrooms: places where learners and teachers come together as groups, in a physically contiguous space, with the goal of advancing meaningful learning. Instead of constraining the learning experience to be narrowly individualistic, this technology supports socially situated interaction and investigation. Moreover, the group itself owns the learning trajectories and the processes of knowledge construction, rather than outside experts or programmers.

Students arrive in class, turn on their devices and find themselves situated in an activity that is not just *about* mathematics, science or engineering but situated *in* an activity that invites them to participate in ways that are meaningful or relevant to STEM-related practices. The network allows artifacts, electronic gestures (e.g., pressing a key) and patterns of interaction to situate and animate the evolving experiences of students. Local actions on an individual device—like, for example, pressing a key to move a simulated character—interact and are coordinated by the network, and are projected, in (near) real time to a public, collective space in front of the classroom.[2]

[2] The implementations of next-generation networked systems may vary, but the top-level design features of the systems are remarkably similar and typically include: individual devices or "nodes"; support for a range of topologies for real-time or near-real time interaction (e.g., peer to peer, peer to group including whole-class, or group to group); wireless flexibility and portability; a core set of meaningful functionality in each device (e.g., at least that of a graphing calculator); and a mixture of public and private display spaces (e.g., the public space can be a computer projection system as with participatory simulations [Wilensky and Stroup 1999a] or a calculator Viewscreen™ with some of the SimCalc materials [Hegedus and Kaput 2003] and the private space can be the students' own individual displays on a calculator, laptop computer, or tablet [Remmler and Stroup 2012]). The network experience is "author-able" in that it allows teachers, students or others to create new activities or change the flow of a given activity. Participants can exchange both group and individual artifacts/data-types including text, strings, numeric values, ordered pairs, lists, matrices, individual and whole class graphs, images and, in some cases, sounds or video.

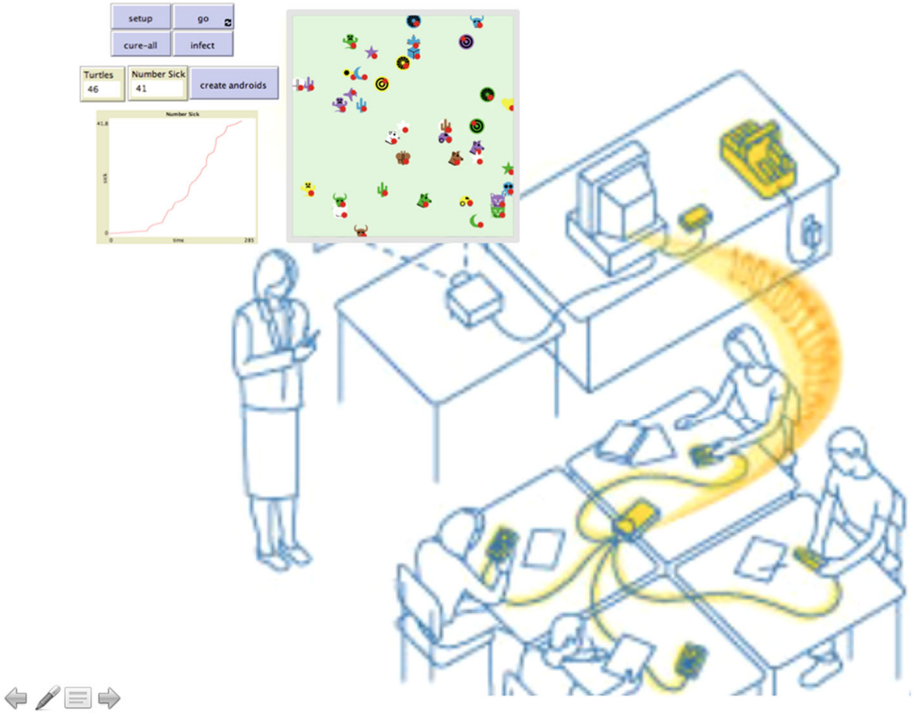

Fig. 2 An illustration of a network-supported implementation of the disease participatory simulation in a classroom

The capabilities of these systems go well beyond the simple aggregation of student responses to multiple-choice questions. Particular attention can be paid to either the students (Roschelle 1992) or the role of the teacher (Mack 2007) or to the interactions between teacher and student (Ares et al. 2009).

Students (Fig. 2) assume iconic roles that are “like the thing” they are modeling. Specifically, the students can *behave* like a rabbit of wolf or like a traffic light, and the relevant behaviors of the system emerge from the interactions (e.g., the relative populations of rabbits and wolves oscillate in ways consistent with fundamental ideas in population dynamics or the traffic becomes more or less congested in ways related to the traffic-related strategies the students implement).

Student participation is also *represented* in the simulations by icons that share features of the role they are assuming: they can look and move like predators or prey, or can be a light positioned at an intersection and turn the color of the light red or green (Wilensky and Stroup 1999a; Stroup and Wilensky 2002). These forms of network-supported interactivity stand to enhance and extend the kinds of role-playing activities that have been used for decades in education either for teaching concepts like the spread of a disease in a population (cf., Stor and Briggs 1998) or system dynamics proper (cf., the *Beer Game*, an inventory management simulation developed, as noted above, by Jay Forrester at MIT in the 1960s and revived by the appearance of Peter Senge’s the *Fifth Discipline* in 1990).

Effort has been expended situating some of these iconic role-playing activities relative to agent-based forms of systems modeling that include the use of powerful tools for agent-based modeling. Network architectures for implementing real-time networked role-playing activities with computers, graphing calculators, or tablet computers are now available (cf., *HubNet*, Wilensky and Stroup 1999c, 2000; *Cloud-in-a-Bottle*, Remmler and Stroup 2012) and significantly enhance the interactivity, replay and real-time data collection capabilities associated with iconic role-playing activities. It is precisely these capabilities, as combined with possible analyses of classroom discourse, that make the HubNet environment particularly powerful for engaging the relations between agent-based and aggregate forms of reasoning for systems learning and teaching.

4 Approaches to Analyses of Classroom-Based Use of Participatory Simulations

Over a number of years we have been investigating students' reasoning about complexity while engaging in participatory simulations. Although we've begun with a presentation of how we currently understand the relationships between aggregate and agent-based ways of reasoning about complex systems, historically and methodologically the development of this position has emerged from many years of iterative involvement with working alongside learners and teachers in (and sometimes out of) classrooms. The task we now attempt to address is to give an account of how we systematized our efforts to make sense of how aggregate and agent-based reasoning do seem to accurately characterize aspects of learner's reasoning, and then how these forms of reasoning interact in relation to the on-going use of network-supported participatory simulations. Then we present our results based on the approaches we've developed with enough detail to allow the reader to evaluate the EC claims in terms of their empirical warrant and/or consider deploying aspects of our approaches or findings in his or her own work.

4.1 Research Questions

From early on in our efforts we have been committed to pursuing a broad set of questions that we then refined and focused for what is reported herein. These broader research questions include the following:

- How do students appear to make use of agent-based and aggregate forms of reasoning to make sense of dynamic systems?
- How are learners' intuitive understandings (and ways of responding), as evidenced in their activity in the participatory simulation, shaped and modified in interaction with agent-based and aggregate modeling environments?
- How does student interaction with each of these two types of modeling environments influence the development of their associated form of reasoning?

In this paper we address a set of more specific questions related to the interaction of agent-based and aggregate reasoning in STEM-related participatory simulations. Our focus is on embedded complementarity as a form of reasoning situated, in this work, relative to the use of network-supported participatory simulations. Some attention is also paid to what happened just prior to, and then after, their use. The research questions for this work include:

- Can we *identify, characterize, and situate* the interaction and development of aggregate and agent-based forms of reasoning in relation to specific STEM-focused activity sequences carried out in classroom settings from within the embedded complementary framework?
- Can we begin to develop and deploy research methods and approaches to *systematize the accounts* of how the interactions and coordination between aggregate and agent-based approaches appear in, and then serve to support, learners' developing understandings of complex dynamic systems?
- How does *participation* within a simulation, where both agent-based and aggregate forms of reference and interpretation are available, enable and advance learners' reasoning about complex dynamic systems?

We look to address these questions in enough detail to allow for extensions of the EC framework by researchers and/or classroom-focused educators.

4.2 Research Sites and Curricular Contexts

This research was conducted in four low socio-economic status (SES) secondary schools in Chicago and Salt Lake City. We worked with iteratively refined replacement units that fit within extant curricula. These two- to three-week replacement units target areas of well-recognized learning difficulty in current curricula (e.g., functions, probability, molecular models of matter, etc.), integrate with teachers' on-going practices and areas of attention, and incorporate successive versions of HubNet capabilities.

4.3 Methods

Due to our ongoing need to inductively develop new theory related to the complementarity of agent-based and aggregate reasoning, we have needed to augment the framework of a design experiment (Brown 1992, Cobb et al. 2003; diSessa and Cobb 2004; Edelson 2002) with some aspects of grounded theory (i.e., Glaser and Strauss 1967; Glaser 1978, 1992, Strauss and Corbin 1990; Urquhart 2000). We have employed a rich array of data collection and analysis to support this effort.

To address these research questions we are structuring our analyses of the project data in terms of the three major forms of analysis associated with grounded theory: *I. Open coding*: we have examined, broken down, compared, and categorized the data especially in relation to the affordances of participatory simulations and the interaction of aggregate and agent-based reasoning; *II. Axial coding*: each category related to the research questions was examined according to the coding paradigm and where knowledge about the relationships between categories emerge; and *III. Selective coding*: core categories were selected from those categories that emerged in the data collection and analysis and from these core categories, themes emerged which have served as the foundation for the grounded theory related to the affordances of participatory simulations and the interaction of aggregate and agent-based reasoning (Strauss 1987; Strauss and Corbin 1990; Urquhart 2000).

To support these forms of iterative coding, the following diverse and complementary kinds of data are being collected: (1) classroom discourse, video-taped, digitized, and annotated using project-developed technologies; (2) artifacts of generative activities including those made available by the capabilities of the HubNet system such as individual key strokes, computational gestures (e.g., jumping one's agent across a screen), student-generated graphs and plots, standard text, multiple choice selection (say, for choosing from a list of programmed behaviors for agents to use during a simulation), data sets, and numerical values; (3) models and analyses students create and (4) video tapes of individual student interviews focused on work carried out during the replace units. We begin our discussion of classroom data with transcribed excerpts from both classroom exchanges and from follow-up interviews.

Given the novelty of the embedded complementary stance as well as the use of network-mediated participatory simulations, the presentation and discussion of these results needed to be substantial. The detail is also meant to respond to current concerns that many recent qualitative research articles, “often provide only very brief excerpts of the qualitative data to illustrate the coding scheme” (Hammer and Berland 2014, p. 38) and that this practice falls well short of “standards for novel methods” first proposed by Schoenfeld in 1992 (p. 181 as quoted in Hammer and Berland, p. 38). We attempt to meet these standards by describing and illustrating our coding methods “in sufficient detail” that readers who “wish to” can apply the methods and by also providing “a body of data that is large enough” to allow readers to carry out analyses “on their own terms” to see if “their sense of what happened” agrees with our accounts (Schoenfeld 1992, p. 181). Then, with the intended meanings of the codes illustrated and clarified from working through these detailed accounts, we are able to step back and present, in tabular form, counts of the relative frequency of occurrences of specific forms of reasoning across different classroom contexts where participatory simulations were used.

To address our research questions, we provide a number of classroom and interview transcripts with embedded researcher memos and subsequent discussions in an effort to show the complementary nature of aggregate and agent-based reasoning and the effectiveness of Part Sims activities in enhancing student abilities to make sense of the emergent properties of complex systems. Although the embedded complementarity stance, as outlined in the first part of the paper, includes having agent-based and aggregate reasoning work together, for coding purposes in these sections we reserved the label ‘embedded complementarity’ for the stronger sense of the integration of aggregate and agent-based reasoning. This stronger sense was discussed earlier as the ‘within’ form of complementary reasoning. The ‘between’ sense of complementarity is to be illustrated by the respective agent-based and aggregate codings being presented alongside side one another within the accounts of using network mediated participatory simulations.

5 Results from Disease Participatory Simulation

To provide enough context for the examples of the different types of student reasoning highlighted below to be understood, we begin with a discussion of agent and aggregate level descriptions related to the Disease participatory simulation. We then move on to analyze classrooms results as well as results from follow-up interviews.

5.1 Agent and Aggregate Components of the Disease Participatory Simulation

As stated earlier, agent-based reasoning typically refers to the rules and strategies of individual agents of a system in relation to the varying emerging properties of a system that are manifested when rules and strategies are altered. In the Disease Part Sim, students are each assigned an agent to control within a grid or field. By controlling the movement of their avatars, the student act as agents in a complex dynamic system. Additional agent icons can be added to the system to examine the effects of increased population density.

Once these “androids” are inserted, they are programmed to wander randomly in the same space occupied by the student avatars.

Rules and strategies for agent behaviors emerge as students gain experience with the activity and the technology. Students can change their step size from one unit up to five units in order to change the speed and manner at which they move through the field. After one randomly selected student is infected with the disease, the disease will begin to spread and students either will attempt to avoid contact or will actively pursue one another in an attempt to spread the infection. Avoidance behaviors are typically, but not exclusively, adopted by uninfected students and can include moving to the periphery of the grid or changing the step size for their avatars. Once infected, students tend to adopt pursuit behaviors. When students comment on these agent-based behaviors and strategies, as well as on the possible effects these will have on the spread of the simulated disease, we will discuss these utterances and related gestures as instances of agent-based reasoning.

As the disease spreads through the population, aggregate data are graphed, in (near) real time, showing the total number of infected agents per unit time. The spread typically results in an S-shaped, logistic or epidemic growth curve. Student aggregate reasoning normally is focused upon the graph and the changing rate at which the disease spreads at different points along the graph (slow at the beginning, fastest near the middle and then slowing again near the end). Other aggregate concepts students can discuss include population density (number of agents per unit space) and the probability of becoming infected when in contact with a sick agent. This probability is one of the parameters that can be changed in the disease participatory simulation and students can predict and then try out what will happen to the shape of the graph if the probability is varied. With the graph of the number sick being calculated and displayed in real time as the disease spreads, and then being available after a trial is completed, the HubNet environment affords students and teachers the opportunity to discuss the relationships between agent-based rules and behaviors (e.g., avoidance strategies) and aggregate concepts (e.g., rates of getting sick) in an integrated, group-oriented, learning context.

In the transcription and coding process we were able to identify a number of incidences of students’ agent-based, aggregate and mutually embedded reasoning about the spread of a disease. To highlight these episodes we present classroom discussion data from a sixth grade science class and a mixed tenth and eleventh grade science class as they explore the dynamics of disease spread through a population. Also included are segments from two follow-up interviews conducted with students from the sixth grade class.

5.2 Sixth Grade Science Classroom Disease Data

The sixth grade class began the unit with a non-computer based simulation activity called *Dice and Disease* (Stor and Briggs 1998). In this activity, after one student is “infected” at random, students contact each other, roll a die, and spread the disease if the roll of the die is higher than a certain value. The activity takes place in phases and the total number of “deer” (students) infected during each phase, or stage, is recorded. The data are subsequently graphed by each student and both the data table and graph are drawn on the board to provide a shared reference for discussion. During subsequent class periods, the students engage in the Disease Part Sim supported by the HubNet environment (Stroup and Wilensky 2002).

Our comments are included in the transcript in two ways. The comments in italics are designed to provide dimensionality by adding context to the transcript (Strauss 1987). Embedded researcher memos highlight various kinds of reasoning. All student names are pseudonyms to preserve student confidentiality. In some cases, an "S" is used to identify a student whose identity could not be determined from viewing the videotape.

In what follows, data from each of these activities as well as from post-unit student interviews are presented. The transcript begins as the class is discussing the meaning of the graphs in relation to the disease spreading through the "deer" population. The teacher has chosen to focus student attention upon the changing "angles" the growth curve makes in relation to the x-axis.

T [*Teacher*]: So what does that tell you about this, the deer? Go back to the deer. What does this say about the deer? These lines, what do they say [*making hand signals signifying a changing angle*]? What is it, what is it trying to say? If I'm going this way [*larger*], what's happening?
S: It increases
T: The deer?
S: More
T: More what?
S: More deer
T: How can you tell?
S: Because, because its getting, cause, cause, when it, when it goes that way its going its getting, wait, less
T: Remember that we're working with an angle [slope of the graph], the angle is what?
S: It's getting larger
T: Four people have their hands up and I'll listen to their answers
Suzie: Well it says that at first there, its like its one like its, its like medium right at first and then it gets larger which means more deer are getting infected and um which are like almost endangered

Aggregate reasoning—The student connects the angle (steepness) of the graph to an increasing number of deer being infected. This sense of number as "more deer" in a segment of the graph is closer to a change happening in time (cf. Stroup 1996, p. 177) rather than as an integrated coordination with time as a rate of change.

T: If you're looking at a graph and I want you to understand how to see that graph how could you tell me that more are getting infected, by looking at a graph?
Suzie: By it getting wider

Aggregate Reasoning—More deer are getting infected when the angle gets wider.

T: What does wider mean?
Suzie: Bigger

In the above conversation, Suzie is able to relate changes in the "angle" of the graph to changes in the number of infected deer. In the following conversation, which takes place immediately after the one above, Suzie is able to extend her thinking to include an explanation for the graph leveling off at the end. The researcher enters into the discussion as a participant observer.

T: So, in the end, what would you like to conclude? What would you like to say before we come to an end? About any of the two graphs. Would you like to make any comments?

Suzie: Um well I, I just have to say that well first one infected deer can cause like a lot then it can get wider but then when they are already endangered, then it can like it gets less because there's no more

Aggregate Reasoning—The student describes the changing slope of the logistics growth curve and explains why the graph changes slope based upon the changing populations of infected and uninfected deer.

T: Okay, and how would the graph look?
Suzie: It would look, first it would look like this [*the angle*], [*hand signals indicating a small angle*], it would look like this, and then like that [*hand signals indicating large angle*] and then like that [*hand signals indicating small angle*]
T: Um huh
R [*Researcher*]: Why does it get bigger at the beginning, when we're going from [*stage*] two to [*stage*] three?
Suzie: Because, because then the, the deer that were already infected infect others but then it gets smaller because they're already infected

Aggregate Reasoning—The foregrounded form of reasoning is the graph and then this is able to be linked to the infection of the deer.

The researcher then introduces the comparative rate related language of "slower" and "faster", instead of "more" or "wider", into the conversation. The students quickly appropriate this language:

R: Right, well but we're now going from here. That's the part we're going to get, this is stage five, right? This is stage five. When it gets slower we have less deer but before that you were saying that it gets faster
Katie: It's getting faster because the more deer that are infected, the more that are going to get infected

Aggregate reasoning—In order to explain why the line is steeper in the middle, Katie uses one aggregate concept ("more deer" infected) to explain another aggregate concept (the "faster" rate of disease spread) in a way that also links the number that are infected with those that are "going to get infected."

Until this point, the conversation has focused upon the prevalence curve (total deer infected over time). The researcher changes the focus to the incidence curve or the number of new cases of infection. The students seem to have little difficulty interpreting this new graph in terms or rate of disease spread.

R: Then it went over here like this, right? [R redraws the prevalence graph as she and the students are talking.] We went over here and now we're paying attention just to how many newly infected deer we have at each stage. Here we had one, that's, that's how we started so when [Researcher labels each stage on graph]. So did I get it right?
S: Yea
R: So now we're trying to think how fast the deer are getting sick, okay. When are they getting sick the fastest?
Suzie: Three, four

Aggregate Reasoning—Interpretation of prevalence graph. The student is able to identify where on the prevalence graph deer are getting sick the fastest.

R: Three four, right here right? [as she circles the two high points on the graph]. So how do we know, before we said that the first graph when the angle is bigger that they're getting sick faster. How do you see it in this graph?

Suzie: It's um higher than the other ones

Aggregate reasoning—Suzie relates "higher" portion of graph to "faster" rate at which deer are getting sick.

R: This is the highest one. So this is the highest? Do you agree with this? And where this is the highest like we had in the [inaudible] we had more deer getting sick cause it's higher so we have the deer are getting sick faster. Right. And when are they getting sick the slowest? Somebody else? When are they getting… Amir?

Amir: Stage five

These student comments illustrate an ability to reason aggregately when interpreting graphical data representing the spread of a disease. More (greater change) is beginning to be integrated with a rate related use of "faster". What seems to be missing from the student comments focused on the graph is the presence of any clear references to agent-based reasoning. The non-network mediated "Dice and Disease" participatory simulation was conducted on a day prior to this discussion. Attending to the graph and the absence of any agent-based cues seem to focus student attention and reasoning towards attending to aggregate based interpretations and explanations.

The transcript discussed next highlights student comments during the Disease Part Sim in the HubNet environment. The first iteration of the simulation has been paused in process. The students and the teacher are engaging in a whole group discussion concerning the events that have occurred during the simulation and are discussing predictions as to what will happen when the simulation resumes.

T: Alright. I'd like to hear some comments, ask you what's happening, why is any happening and what do you think will happen if we continue or will nothing [inaudible]

Open Ended Question—Used to invite observations, interpretations, explanations and predictions.

Jodie: Every time you touch the red dot you get infected

Agent-based observation about how the disease is spread—Jodie identifies the mode of transmission—how it works—using an agent-based rule based on "touch" or local contact. Also significant is the phrase "every time" as a way of referring to the likelihood or probability (100 % in this case) of getting disease with each contact. This rule for interaction is unlike the local rule use in the non-network based disease simulation where infection did not happen "every time" there was contact, but was instead based a roll of the dice yielding a probability of infection less than 100 %.

T: Every time you touch the red dot you get, got infected, someone got infected. Alright. Emily?

Molly: I realize that as people were trying to go up and the graph was growing and people were kind of [*inaudible*] but the people who were on the side weren't getting infected because the graph wasn't going straight up. [*inaudible again*]

Embedded Complementarity (EC)—Agent-based informed by aggregate reasoning. Molly's agent-based reasoning about kinds of movement and strategies deployed for avoiding *infection is informed by her aggregate reasoning that the number of infected is not increasing. Molly integrates kinds of agent-based avoidance behaviors exhibited by the icons with changes in the graph—growing but not going "straight up"—that are aggregate-base ways of attending to what occurs. In this way she is connecting agent-based behaviors to aggregate changes in the population of infected individuals. Reasoning in one way is bound up with, or embedded in, reasoning with the other way of reasoning.*

Barry: If one of those things…
T: Turtles? [The agents in NetLogo language are sometimes referred to as 'turtles.']
Barry: Yea. If one of them gets infected then [*some will get infected*], but some of them, some of them won't

Prediction about the totality of spread of the infection in the population. If one gets infected, most, but not all, will get infected.

T: That's what you're predicting?
Barry: Yea
T: Okay. Sam
Joe: [Inaudible. Something about turtles going around]
Steven: I know. They teamed up. It's like they, the ones that were infected. It seemed like it

Agent-based observation—An agent-based strategy of individuals being "teamed up" is proposed.

[Large uproar from the class. Lots of overlapping speech.]

Katie: Everybody was going up once everybody was getting infected …
T: I can't hear Katie. [Pause]. I'd like to hear Katie
Katie: Um almost everybody at the bottom got infected so everybody was going up and it looked like they were all teaming together to go up and [inaudible]

Agent-based observation—Describes teaming strategy, but in more detail. Teaming up is a named emergent phenomenon with related implications.

T: Suzie
Suzie: Um well I was going up and then um I didn't notice what was happening with the graph but it seemed that everybody was teaming up because I'm surrounded by all these NU's ["NU" was one of the randomly assigned icon shapes] and everything [pointing to screen showing the paused simulation and the current position of the agents]

Agent-based observation—teaming up strategy but adds that she could not pay attention to both her movements and the graph.

Steven: Right those NU's got it
Jamal: Its like ah you can see its like all of them mostly on the right got infected and like there's one ah, there might be like one left and they're probably going to team up on it or run to the negative

Agent-based observation and related prediction.

The above conversation illustrates the effect of features of the HubNet environment upon how students talk and reason about the spread of a disease. Whereas with the prior use of the *Dice and Disease* activity student comments reflected primarily the use of aggregate reasoning to make sense of the data, after participation in the HubNet-based activity, agent-based reasoning begins to play a significant role in the conversation. In addition, as evidenced by Molly's comment, students begin to overly inter-relate agent-based and aggregate phenomena.

The series of transcripts presented next are of the class discussion that took place after the second simulation of the class period. During the first simulation, the show-ill button was activated so all infected agents were marked with a red dot. In the second simulation, the show-ill was turned off, so that no students could see who was infected in the upfront space. On their individual devices, however, they would be informed when they became infected. The discussion shown below focuses on comparisons of the agent behaviors and aggregate results from two simulation trials.

T: Alright. What happened? [*Students call out several answers*.] Terika?

5.3 Open-Ended Question

Terika: You could know that you were infected if you touch something and ah the connecting thing [*graph*] goes up and if your not infected and something touches you it goes up

Makes connection between agent-to-agent contact and increases in the graph (and not the message that would appear on her device). This shows that she is able to attend to both agent-based and aggregate aspects in this environment.

T: So that's how you told, I mean that's how you could tell? Alright. Kiran?
Kiran: I think that the population of people getting infected got a little less [she references "population" and "less" but we believe she is referring to a decrease in rate and not number] because the people didn't know who was infected and they ran away so people couldn't infect them that much. But then the people that were already infected they didn't feel that they infected anybody but I [inaudible]

Embedded Complementarity—Aggregate reasoning informed by agent-based reasoning. Agent-based explanation for the difference in the aggregate results between the show-ill and hide-ill simulation trials.

Later in the same discussion, Kiran introduces the concept of rate into the conversation.

T: …You're saying that it's more like a steady? What do you mean by steady?
Kiran: Like people are getting infected in a certain [inaudible] beginning at a certain rate but the last one people weren't getting infected at a certain rate
T: Okay. Anyone else have a comment?
R: [inaudible]….summarize. I'm sorry, your name is Karen?
Kiran: Kiran
R: Um, I would sort of like for [inaudible] saying, because you were saying something interesting you were talking about rates, remember our discussion at the end of the last lesson and you were talking about how flat it is, how its going up, or if its slanted or not. How is this related to rates? [inaudible]…make sense of this graph thinking about rates, how fast people are getting infected
Ahmad: Um since the first one that we did people knew that they were infected so people moved to get away from them

Agent-based reasoning relating to movement.

R: Ummmm

S: And the people who knew they were infected they were going after the other ones

Agent-based strategy for "people" who did know they were infected.

R: Wow! …Interesting. That means that these trials aren't happening at the same rate
S: For the second one nobody knew who was infected so they were all staying away from each other so in the beginning nobody, no one knew who was infected and so they were all away from each other. Then Michael said that he was the one and everyone started to get away from him [inaudible]

EC—Aggregate informed by agent-based reasoning—The student uses the different agent-based behaviors, which were influenced by the hide-ill/show-ill conditions, to explain the differences in the rates at which the infection spread between the two simulation trials.

R: …So what do think, are people getting sick faster or slower when you don't know who's sick
[Several students call out "slower".]
S: Slower because everyone's kind of staying away from each other cause they don't know if they are infected or not. So it takes a while for everybody to get infected

EC—Aggregate informed by agent-based reasoning. The student uses the agent-based strategy of staying away from other people as an agent-based reason to give and account of the aggregate phenomenon of a disease moving more slowly though a population where you can't see who is sick.

Steven: Even if they are infected when they do get together they're both already infected So there's a lot of re-infectees

EC—Aggregate informed by agent-based reasoning. Steven uses the agent-based phenomenon of the increased likelihood ("a lot of re-infectees") of contact between two already infected agents having the aggregate effect of causing no increase in number of infected agents, thereby leading to a slower rate of disease spread.

R: Okay
Steven: Just like in the dice game. (A reference to the previous day's use of the Dice and Disease activity)

The student is drawing connections between current simulation and the previous activity.

In addition to the increased attention to agent-based phenomena mentioned earlier, after just two simulation trials using the HubNet environment, the students exhibit the ability to reason using both agent-based and aggregate levels to make sense of the spread of a disease through a population. Furthermore, the students are able to compare and contrast results from two different simulation trials under different conditions (showing or hiding who is sick), make aggregate predictions about future simulations in the light of alternative agent-based rule sets, and interpret two examples of rate-based graphical data using both agent-based and aggregate concepts. It is important to note that the students displayed aspects of what is meant by embedded complementarity discussed earlier: agent-based informed by and/or situated within aggregate reasoning and aggregate informed by and/or situated within agent-based reasoning. It is also interesting to look at the data presented longitudinally. During the Dice and Disease activity discussion, the students used predominantly aggregate reasoning. Then initially, during the Disease Part Sim, the students focused primarily upon agent-based reasoning. During the discussion after the second disease simulation, student comments predominantly indicated the complementary nature of agent-based and aggregate reasoning.

5.4 Tenth and Eleventh Grade Science Classroom Disease Data

Data analysis of student comments in the tenth and eleventh grade science classroom running the Disease Part Sim yielded similar results. Below we present transcripts from a 10th and 11th grade science class after running a few simulations. The transcript begins while a simulation is in process. For the first time, the students are running a simulation with the "show-ill" turned off.

Rosie: I think that I am infected because whenever I hit somebody it goes up
Freddy: So do I. I stepped on the blue truck, ah the blue boat and the scale went up

EC—Makes connection between agent-to-agent contact and increases in the graph. Shows that students are attending both to the agent and aggregate aspects of the network-mediated participatory simulation.

During the same simulation, the following discussion occurs:

Freddy: I think this is going to go slower

Aggregate prediction related to class hypothesis for this simulation.

T: Could be
Rosie: I think that's just because they like to go after each other when they are infected (a likely reference to previous trials when show-ill was enabled)

EC—Aggregate informed by agent-based reasoning. Rosie uses agent-based strategy/behavior related to being able to see in previous trials who is sick and act on this to support Freddy's aggregate prediction that when this is no longer possible it will "go slower".

[Short time passes with a few other comments.]

S:	Who's infected?
R:	There's four more people to go, two just got, so there's only two more now. One more
Sheldon:	Hit the infect button so we know who it is
Freddy:	Click on the infect button real fast just to see who's all infected
Jakey:	The infect button is on, he has the show ill off
Freddy:	Show it on real fast just before we kill them off
R:	We could do that
Freddy:	Yea, go ahead
Students in Unison:	Yea!!! [In reaction to changing show-ill to the 'on' position]
Rosie:	This is exactly why it takes longest 'cause you'll go after whoever is not sick

EC—The aggregate number sick indicated to the students that there are "only two more now" who are not sick. The move is to alter the local agent rules so as to then be able to infect the remaining two and thereby have the aggregate number sick reach the population number.

5.5 Sixth Grade Class Interview Data: Post Disease Simulation

The research team carried out post-activity, semi-structured interviews with ten students selected randomly from the sixth grade classroom highlighted previously. Researchers conducted the interviews after the class participated in the unit on disease. The results of these interviews indicate that students retained the ability to use both agent-based and aggregate reasoning to make sense of situations, data, and graphs. The first interview transcript segment is with Steven who made several comments in the classroom transcripts presented previously. (The "I" indicates interviewer).

I: How did it happen and how might it not happen, that everyone would be infected?
S: It might happen and, they um, people might just want to spread it. They'd be like oh, this person is um done this to me I want everyone else to suffer with me

Agent-based reasoning used to give an account of the rationale for behaving in a way that results in the disease spreading throughout a population.

I: Um huh
S: It might not happen when in um, "I'm sick, I'm sick. I don't want to, I don't want anyone else to get sick. I'll just stay by myself

Agent-based reasoning ("stay by myself" or self-imposed quarantine) used to explain how to avoid ("might not happen") having the disease spread throughout a population.

I: So when you were playing with the calculators (in the HubNet system) when you got sick, what did you do?
S: I, I tried to stay away from everyone

Agent-based strategy/behavior for not-sick agents informed by the expectation while the sick agents could choose to stay by themselves, they don't and their pursuit of not-sick agents provokes a "stay away from everyone" strategy.

I: When you were sick [surprised]?
S: Yea. I changed, ah, I changed my step number sometimes to like five so as someone would come close to me [inaudible] I would just go up, so I wouldn't infect'em

Agent-based avoidance strategy at odds with the pursuit strategy enacted by other infected agents. S increased the step size of his agent (from the default of one, to "like five") in a way intended to make it so he "wouldn't infect'em." Variant strategies emerge in relation to alternate intents for the spread of the disease.

I: Was everybody doing that?
S: No. Some people were infecting on purpose
I: They were infecting on purpose?
S: Yea
I: Um huh, and…
S: When they didn't know they were infected they were, they were just trying to stay away from everyone, but that didn't work because, um there really wasn't enough room for everyone to stay away from each other

Agent-based reasoning linking the pursuit (for sick) and avoidance (for not sick) strategies. Then with show-ill off the idea of limited space is used to explain the spread of the disease.

I: Okay, so, so if we made the screen bigger…

S: The grid bigger?
I: Yea, then what would have happened?
S: Everyone would have just stayed in this area, including the person who was infected. If they, when they knew they were infected they might go after them, it depends on how the person feels, ah, the deer feels

Agent-based reasoning—Change in grid size can change agent behaviors. Some behaviors, however, may be taken up again—e.g., the pursuit or "go after" strategy—in a way that references "how the person feels." The simulation allows for personal agency related to normative intent for the spread of the disease.

I: Um huh. Okay, so um what if ahhh what if even more kids like say um a whole 'nother class of kids, we had a ton of calculators and just everyone was doing it on the same sized screen, what would happen? More kids. Twice as many kids.
S: On the same grid that we were just talking about?
I: On the one that was in class. I mean does it matter? It matters? Why does it matter?
S: It matters a lot, because there's um, there's probably more, more, there's more chance because there's less room to move and probably they will have the people who were on top of each other like they were covering one another, when they're covering one another the disease just spreads
I: Um huh
S: It just spreads like all three of them would get it at the same time. Or four, however many. And they would get it at the same time because such and such person bled on them and their blood contains the disease, their blood contains the disease. And when their blood gets inside their bloodstream they, not I wouldn't say all of them would automatically have it, but almost automatically have the disease because its hard to get the disease out

EC—Expectations regarding the aggregate behavior of the system is linked to implications at the agent-based level of having more agents in the same amount of space. Crowding could even become so pronounced that it would result in having multiple agents on top of each other. This would mean "three" or "four" would "get it at the same time" with aggregate implications for how the disease "spreads". The uses of "automatically" and "almost automatically" implicate expectations related to likelihood, or comparative probabilities, for what would happen in this more crowded context. This sense of how automatically the spread would happen is coordinated with a degree of difficulty in responding to the spread of the disease: "…it's hard to get the disease out."

In the transcript above, Robert begins by foregrounding agent-based reasoning in the form of behaviors and strategies of the "deer", but these agent behaviors are seen to be closely tied to specific overall implications for the spread of a disease. For example, in his last comment, he introduces a number of aggregate concepts including probability (i.e., how "automatic"), effects of increased population density (more deer in the same space), and increased rate (three or four "at the same time") of disease spread. In this comment, he combines an array of aggregate concepts with specific agent-based behaviors like "they're covering each other" and "such and such person bled on them" to reason about the effects of an increased number of agents in a system on the spread of the disease. By integrating and linking these aggregate and agent-based notions, he displays aspects of the complementarity of these types of reasoning that we designate, in an overall sense, by the use of embedded complementarity.

The next interview highlighted is with Suzie. As in class (see above), Suzie demonstrates an ability to use and integrate both aggregate and agent-based reasoning when discussing the spread of a disease.

I: How did the step size change things?

S: The step size changed things because you, like say this is you and there was an infected person coming to you and you had all these other people infected. If you were on one step, you'd also get infected because then the person was right there. But with the step size you leaped from that person, you can get out so it increased your chances of not getting infected

EC—Aggregate informed by agent-based reasoning. Step size of five (agent based aspect of the strategy) resulted in a decreased probability (aggregate concept) of getting sick. Similarly, having step size set to "one step" resulted in a higher comparative probability of getting sick.

I: Okay. And um, so, so what happened, for the spread of the disease then?

S: Well it would slow down because I think it ah it slowed down because then when the step size changed people moved around more and yea you could have got infected by other people when they moved around more but they stayed away from certain people a longer time than if it was a step size one

EC—Aggregate informed by agent-based reasoning. Rate of disease spread slows due to a change in agent-based behaviors and strategies. Larger step size would, on balance, result in staying "away from certain people longer" and the spread of the disease would be "slowed down".

Later in the same interview, Susie is asked to come up with other factors that would decrease the rate of disease spread.

I: Okay so what, so what's something you could do to make it spread slower?

S: You could um, whelp first of all you could make it like a more open space I guess and you could make it well I mean I don't know if this really would work but you could also make it that um they're like well if you get the disease you die so then you die so then like not other people like if you die then other people won't be able to get from you 'cause you're dead. So, that I mean unless they eat you but I don't think that will happen. Um and then that's…

I: So you die and you wouldn't be able to move?

S: No you wouldn't be able to move to infect other people. So if they see you die the deer, I mean if they were real deer, if they see you die then the deer will know the disease so they could move around, like get out of the area

I: Um um. So, so you're saying if, what if, if, deer knew they were sick and you now decided to not move?

S: Then that would increase the chances of them not spreading ah like a lot because then if you decided to move and no one came near you, then they wouldn't get infected. So then I mean, you would still be infected but they wouldn't so the population would still not be all sick

EC—Coordinating implications. With the goal of slowing the spread of the disease Susie is able to pick out a number of changes that would have this result and does so in a way that can be linked back to this goal. Within a coherent framework for impacting the outcomes she integrates aggregate and agent-based reasoning.

Taken together, Susie's previous three comments are indicative of the complex modes of reasoning required to understand the dynamics of a complex system like the spread of a disease in a population. She first suggests that creating more open space will result in a decreased rate of disease spread, presumably, as this will result in less chances for agents to have direct contact. She then suggests adding mortality into the system having the effect of inhibiting the movement of agents resulting in a slower rate of disease spread. She links mortality to the disease spread, especially when the mode of transmission is agent-to-agent contact that requires movement. If an agent stops moving this means the movement could only come from the other agent, and even this possibility could be reduced due to the increased likelihood of that agent avoiding the infected deer. She also mentions the agent-based behavior or strategy of an infected deer not moving resulting in the other deer not being "near you" and thereby stopping the spread of the disease. Suzy is therefore able to integrate both aggregate and agent-based scenarios that would result in a slower rate of disease spread.

Towards the end of the interview, Suzie makes a significant comment illustrating her ability to interpret a rate-based graph.

I: Um, if someone did a really good job of staying away from people, what would that do to the graph?
S: It would, it would sort of make it go not to up it would sort of be more horizontal but it, you're, still one person only so and there's a whole population of deer so it wouldn't affect it that much but it would go only down just a bit, just a bit
I: But it would go down
S: Yea. Not really down. It would like go horizontal, straight

Her answer to the first question is yet another example of aggregate informed by agent based reasoning. Her last comment however, catches her (and the interviewer) in a mistake. She realizes that the agent-based avoidance behavior will not result in the graph going down, but only in the stabilizing of the graph in a horizontal direction. Apparently, she is able to reason that without mortality, the number of infected will always remain either the same or increase, but never result in a decrease in number and therefore never a decrease in the graph. Such sophistication in reasoning about variable rate transcends what is supported in most sixth grade math and science curricula. The use of the participatory simulation seems to support the development of nuanced forms of rate-related reasoning.

6 Results from Gridlock Participatory Simulation

The analysis of using the Gridlock simulation includes data from a tenth and eleventh grade algebra classroom. A sixth grade social studies classroom also conducted a unit using the Gridlock simulation.

6.1 Agent and Aggregate Components of the Gridlock Participatory Simulation

In Gridlock, the students operate traffic lights in a city that is set up like a grid with streets oriented north–south and east–west. The dimensions of the grid can be altered to fit the number of students in the class so that each student can operate a light. Up to 200 cars can be introduced into the system and the speed of the cars can be varied. The traffic flows in a one-way fashion on all streets resulting in cars going both south and east. The students have two options in operating their lights: their light can be green for south bound traffic and red for east bound traffic or vice versa.

Two types of agents inhabit the Gridlock system: cars and lights. Cars can travel slow, medium or fast along a sliding scale resulting in one type of agent-based behavior. In addition, students can change their individual lights at random, or lights can be changed only when a certain number of cars back up at a given intersection, or students can somehow coordinate the changing of their lights. Regardless of which approach is used, students can use these agent-based strategies to make sense of emergent results of the system. We call these types of comments agent-based.

Aggregate results are simultaneously displayed on three graphs: stopped cars, average speed, and average wait time. The class can choose to show any one of the graphs or all three simultaneously. The last option is often chosen so that students can compare the graphs and decide which metric is most useful in identifying "good" (or bad) traffic flow.

6.2 Tenth and Eleventh Grade Classroom Gridlock Data

The transcript presented below takes place in an eleventh grade algebra class on the second day of running Gridlock. The class is comparing the results of a simulation just run and a simulation run on the first day. The students have chosen the average wait time of the cars as their preferred metric. Lower wait time is preferable. As a way of better managing the traffic flow the students have begun to explore the use of a strategy of keeping the direction of the flows 'in sync' and changing their lights together at regular intervals (e.g., every 10 s). Lights being in synch, or out of sync, become salient for their analyses.

T: Alright
Rosie: It dropped. The wait time dropped dramatically from last time
Freddy: Yea that's…
Rosie: The wait time's a lot less cause where…
T: [*Interrupting. Overlapping speech.*] Yyyyea the wait time went way down at some point in that simulation
Rosie: Well, where, where it spiked up that's about where it was last time, like just constantly it was up, up there and now its…. Well I think that spike is when um, six went of sync and…

EC—Aggregate informed by agent-based behavior. When "six went out of sync" the wait time graph "spiked".

Freddy: ah, six…
Rosie: …and 18 went off sync one time, and 14 went off sync at one point
Freddy: And three

In these discussions, Rosie makes a connection between the aggregate phenomenon of an increase in wait time to a change in agent-based behaviors of the lights as they went out of sync or "off sync" at "one point". After running another simulation, the students are able to make similar connections.

Terry: I think the main reason is that it [average wait time] went up that high was because my stoplight wasn't responding

EC—Aggregate reasoning informed by agent-based behavior. The student relates a non-functioning light (agent-based) to an increase in overall wait time (aggregate based).

More than was true for the disease participatory simulation, one agent behavior can have a significant overall effect on the simulation.

Rosie: [overlapping speech] The number of stopped cars is, is pretty similar to the graph we had yesterday
T: Actually yea it's getting a lot closer to yesterday. I was ah, especially at the beginning over here
Karen: That is so cool, it was like
T: Um huh
Rosie: Well average wait time went up just like it did every other time we started…
Freddy: Squiggle, squiggle, squiggle [oscillations in graph]
T: That's true
Rosie: As we go along with ten seconds it always goes up

EC—Aggregate informed by agent-based reasoning—Relates the agent-based strategy of changing lights every ten seconds (agent-based strategy) to an overall and gradual increase in average wait time (aggregate based). This means the traffic flow is getting worse.

T: It just keeps getting higher and higher
Rosie: So I seriously think if we did every five seconds cause it, it goes, it will come up like it goes up for ten seconds and then when we hit five it goes down
T: Um huh
Rosie: If we did it every five seconds it would basically be lower and be more not going, it would be more like

EC—Aggregate informed by agent-based reasoning—Based upon previous aggregate outcomes of the ten-second strategy, Rosie is able to make a prediction regarding the aggregate results of a change in agent-based strategy (switching the lights every five seconds). The two forms of reasoning are coupled.

As in the disease data highlighted previously, the students may start off appearing to use aggregate-informed-by-agent-based reasoning in the Gridlock discussions to make sense of aggregate results. The students focused upon the average wait time graph and searched for, and used, agent-based behaviors and strategies that accounted for the aggregate results that they observed. Their explanations, however, suggest that their reasoning about how the system works has the two forms of reasoning interrelated even as it may appear to foreground the aggregate behavior in starting the explanations. The close or near-immediate coupling is what has us coding the reasoning as EC. This contrasts somewhat with what was observed for the disease simulations where it was easier to find instances with one kind of reasoning foregrounded for a given segment of reasoning. For the Gridlock Part Sim, the macro and micro forms of reasoning were, apparently for these students, more conspicuously bound up in one another.

7 Comparative Results from Data Analyses

Based on the kinds of coding discussed in the previous sections, Table 1 shows overall results of from our data analyses including the number of activities, classroom sessions, comments coded, and the relative occurrences of agent-based reasoning, aggregate reasoning and embedded complementarity. The table serves to illustrate the scope of our efforts to examine the interaction of agent-based and aggregate forms of reason. The data is

Table 1 Instances of kinds of reasoning for two participatory simulations and subsequent interviews

Activity/setting	Number of class/interview sessions	Coded teacher/student comments or questions	Agent-based reasoning	Aggregate based reasoning	Embedded complementarity
Disease	8	633	13	26	11
Gridlock	10	263	18	4	6
Student interviews	3	65	11	6	11
Total	21	961	42	36	28

from the four separate classrooms discussed above: a middle school science class, a middle school social studies class, a high school science class and a high school algebra class.

Table 1 also compares the incidence of teacher/student comments in general to the incidence of the forms of reasoning that are our primary focus. We were cautious in what we were willing to identify as instances of agent-based, aggregate and embedded complementarity (of the stronger, 'within,' form). The incidence of these comments was about 11 % of the overall total. The kinds of reasoning we are most attentive to, then, should be understood to exist with, and alongside, other kinds of comments and questions.

Note, too, that while all three forms coded for are present in all of the activities/settings, there is significant variability in relative amounts. As might be expected, some activities seem to foreground particular forms of reasoning more than others. There is no determinate mix or balance of the respective forms of reasoning, which suggests that the instances and interactions should be allowed to emerge and develop as they will in relation to specific activities, contexts, and lines of reasoning advanced by the students and the teacher.

8 Discussion of Results

The results from the empirical section of this paper advance a systematized account of how aggregate and agent-based reasoning are present and interact in relation to the on-going use of network-supported participatory simulations. These efforts were to serve the goal of advancing learning about complex dynamic systems particularly in the context of learning and teaching STEM domains. The challenges of developing approaches for investigating learning in group-based settings are, of course, increased first by the use of a new network architecture for supporting the implementation of participatory simulations, and then by the novelty of the embedded complementarity claim relative to systems related reasoning. We sought to work across a number of grades and subject areas and integrated aspects of analyses of classroom interactions and discussions augmented by in depth interviews and an effort to count instances of kinds of reasoning exhibited during the respective implementations of activities and the interviews.

To address the first of the three more focused research questions, introduced at the beginning the empirical analyses, in relation to specific STEM-focused activity sequences carried out in classroom settings and within the embedded complementary framework, we have presented a substantial set of examples of where we were able to *identify, characterize and situate* the interaction and development of aggregate and agent-based forms of reasoning. Whether on not one may want to subscribe to all aspects of the relationships we advance in the introductory portion of the paper or in this empirical section, we would suggest that the case for both kinds of reasoning being identifiable and significant in characterizing learners' ongoing sense making is strong. Many elements of what was said and done by the students in relation to the activity were made tractable and mutually informing when viewed through the lens of relating agent-based and aggregate forms of reasoning.

In response to the second research question, we were able to use the codes denoting the relative foregrounding of aggregate, agent-based or (the stronger form of) embedded complementarity to generating overall counts of instances in the respective activities or interviews (see Table 1). While the reader will have to judge the coherence and/or clarity of these efforts, we do believe we can use these frameworks to *systemize* efforts to characterize the interactions of aggregate and agent-based forms of reasoning.

Worth noting relative to this effort to systematize the analyses, we have occasionally had rather significant discussions among our research teams about which of the three codings 'really' applied to a particular segment and/or how or when one form of reasoning interacted with the other. Similar discussions have emerged when earlier versions of this paper were shared with colleagues or reviewers.

As much as these disagreements might serve as a caution, or even an admonishment to do better going forward, in terms of the larger claim related to the 'embedded complementarity' of aggregate and agent-based forms of reasoning, none of these exchanges have been about the forms of reasoning being in any sense 'incompatible' or necessarily working against one another. Indeed the fact that it has proven to be a challenge, sometimes, to clarify precisely which form of systems reasoning is foreground or how, or when, it is that one leads into, or is already co-incident, with the other, might serve as further evidence (of a sort) that these forms of reason can, and often do, work together in ways that can frustrate even our best efforts to parse them adequately for coding purposes (a similar sense of coding disagreements being a kind of data is discussed in Hammer and Berland 2014).

Yes, as was noted at the beginning of the paper, agent-based and aggregate forms of reasoning are distinct. And, hopefully, at least some of the examples provided above do illustrate how one or the other might well be foregrounded in a particular case, episode, or context such that it can be coded either (predominantly) agent-based or (predominantly) aggregate. Moreover, there is value for both the students and the teacher in staying attentive to how they are different and how a particular idea can be most thoughtfully addressed within one form of reasoning. Yet, after carefully following the examples discussed in the empirical section, the case for the complementarity in the ways highlighted in this paper should seem at least credible.

The last of the three research questions focused on how the *affordances of participatory simulations*, particularly as implemented in the HubNet architecture, makes visible, and supports, learners' effort to reason about complex dynamic systems in ways consistent with the embedded complementarity account. In many of the cases reported above the features of the environment and activities proved "ready to hand" or readily accessible to the learners to help them express ideas or scaffold the development of their thinking. This transparency in use also made it possible for the teacher and/or research to better understand what the students were saying, doing or intending. Not only does this suggest these sorts of activities, as supported by this kind of group-oriented architecture, are useful but it also lends support to larger efforts to view STEM-related teaching and learning as forms of participation and emerging agency in relation to communities of practice.

9 Concluding Remarks

Consistent with a broad set of efforts since the sixties to advance systems learning and/or complexity theory as a form of expressive literacy for all school aged learners we have introduced, explained and illustrated embedded complementarity as an integrative stance for relating two major forms of system-related learning and reasoning. The agent-based and aggregate approaches are, as we have noted, distinct yet we outlined how there are forms of complementarity *between* and *within* these approaches when analyzing complex dynamic systems. Then we sought to explore the ways in which embedded complementarity can be used to analyze how learner understandings progress in STEM-related participatory simulations supported by the HubNet learning environment. We coded for aggregate, agent-based, and mutually embedded forms of reasoning to analyze transcripts and follow-up interviews from the classroom-based use of the Disease and Gridlock Part Sims.

We found that the learners used and built on the interdependence of agent-based and aggregate forms of reasoning in ways consistent with the discussion of embedded complementarity outlined in earlier parts of the paper. Throughout, it was our intent to provide sufficient detail and instances of coding such that researchers and/or classroom based educators might be able to attend to, analyze, and advance complex dynamic systems related learning and teaching, especially in STEM related domains.

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